PROBLEM SET 6

DUE MAY 10

1. Regular problems

1.1. Let k be a field. Show that $k[x^2] \subset k[x]$ is an integral extension.

1.2. Let B be an A-algebra and let $y \in B$. Show that if y is integral over A, then the subring $A[y] \subseteq B$ generated by A and y is finite over A.

1.3. Let B be an A-algebra and let C be a B-algebra. Show that if C is finite over B and B is finite over A, then C is finite over A.

1.4. Consider the complex numbers $\mathbf{C}$ as an algebra over the integers $\mathbf{Z}$ in the obvious way. A complex number $\alpha \in \mathbf{C}$ is called an *algebraic integer* if it is integral over $\mathbf{Z}$. Prove that the algebraic integers form a subring of $\mathbf{C}$.

1.5. Let k be an infinite field and let $p(x) \in k[x]$ be a non-zero polynomial. Show that there exists $\alpha \in k$ such that $p(\alpha) \neq 0$. Find a counterexample to this statement dropping the assumption that k is infinite.

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