Math 748 Homework 3

Due Wednesday, September 27

1. Let $K \subseteq L \subseteq M$ be separable extensions of fields. Show that $\mathrm{Tr}_{M/K} = \mathrm{Tr}_{L/K} \circ \mathrm{Tr}_{M/L}$ (a precisely similar argument shows that $\mathrm{N}_{M/K} = \mathrm{N}_{L/K} \circ \mathrm{N}_{M/L}$, but you only need to do the trace version).

2. Let L/K be your favorite non-separable extension (say explicitly what it is). Find $\mathrm{Disc}(L/K)$.

3. Find the ring of integers of the number field $\mathbb{Q}(\alpha)$, where α is a root of the irreducible polynomial $x^3 + x^2 - 1$.

4. Show that if α is a root of $x^3 - x - 4$ (which is irreducible), then $\mathbb{Z}[\alpha]$ is not the ring of integers in $\mathbb{Q}(\alpha)$ [Hint: consider the minimal polynomial of $\alpha(\alpha + 1)$.] Find an integral basis for $\mathbb{Q}(\alpha)$ (be sure to prove that what you have found is indeed an integral basis).

5. Find the ring of integers of the number field $K = \mathbb{Q}(\alpha)$, where α is a root of $x^3 - 3x^2 + 2$ (which is irreducible). What is the prime factorization of the discriminant Δ_K of this ring of integers?