Math 748 Homework 5

Due Wednesday, October 11

1. Give an example of an integral domain B, a non-zero prime ideal $\mathfrak{p}$ in B, and a subring A of B such that $\mathfrak{p} \cap A = 0$. (Note that by a result from class this can't happen if B is integral over A.)

2. Let $\mathfrak{a}$ be a non-zero integral ideal of a Dedekind domain B. Show that in every ideal class of $Cl(B)$ there is an integral ideal relatively prime to $\mathfrak{a}$.

3. Show that every ideal $\mathfrak{a}$ in a Dedekind B domain can be generated by two elements, i.e. there are $a, b \in B$ such that $\mathfrak{a} = (a, b)$ (which is the same as $aB + bB$). Do not use localizations in your proof.

4. Let $d \in \mathbb{Z}$ be squarefree and p a prime number number not dividing $2d$. Let $K = \mathbb{Q}(\sqrt{d})$. Show that the ideal $(p) = pO_K$ is prime if and only if the congruence $x^2 \equiv d \bmod p$ has no solutions.