Math 748 Homework 7

Due Wednesday, October 25

1. Find the class numbers of $\mathbb{Q}(\sqrt{-163})$, $\mathbb{Q}(\sqrt{-10})$, and $\mathbb{Q}(\sqrt{14})$. If you think an ideal is non-principal, be sure to prove it.

2. Show that $\mathbb{Q}(\sqrt{-23})$ has class number 3.

3. Let K be a number field. Prove that there is a finite extension L of K such that for every ideal $\mathfrak{a}$ of O_K, the ideal $\mathfrak{a}O_L$ is principal. (Hint: use the finiteness of the class number.)

4. (a) Show that $K = \mathbb{Q}(\sqrt{-1}, \sqrt{5})$ is unramified over $\mathbb{Q}(\sqrt{-5})$. (Hints: think of K as an extension of $\mathbb{Q}(\sqrt{-1})$. You may also want to use the number field version of the factorization theorem, i.e. if $L_1 \subset L_2$ is an extension of number fields and $\mathfrak{p}$ is a prime in O_{L_1}, then the factorization of $\mathfrak{p}O_{L_2}$ can be obtained by factorization of the appropriate polynomial in $(O_{L_1}/\mathfrak{p})[x]$.)

   (b) Can there be any other unramified extensions of $\mathbb{Q}(\sqrt{-5})$?