A brief synopsis of $\mathbb{C}$

- **Basic definitions.** The *set of complex numbers* $\mathbb{C}$ is Euclidean 2-space $\mathbb{R}^2$ equipped with *vector addition* and *amplitwist multiplication*:

$$(a, b) + (c, d) := (a + c, b + d),$$
$$(a, b) \cdot (c, d) := (ac - bd, ad + bc) = \begin{pmatrix} a & -b \\ b & a \end{pmatrix} \begin{pmatrix} c \\ d \end{pmatrix}.$$

In particular, complex multiplication has the same effect on the plane as multiplication by an *amplitwist* matrix.

- **Complex plane.** We refer to $\mathbb{C}$ as the *complex plane.* We view $\mathbb{R}$ as the x-axis in $\mathbb{C}$ by identifying the real number r with the ordered pair $(r, 0)$. We refer to this as the *real axis.* Similarly, the y-axis is referred to as the *imaginary axis.*

- **Complex numbers.** We denote by i the complex number $(0, 1)$, which satisfies $i^2 = -1$. Thus, every complex number can be written as

$$z = (a, b) = a + i\,b = a + b\,i.$$

The number a is called the *real part* of z, denoted $realz$; the number b is the *imaginary part*, denoted $\operatorname{Im} z$. Hence, every complex number can be written as

$$z = (\operatorname{Re} z) + i(\operatorname{Im} z).$$

- **Modulus.** The modulus of a complex number z is the Euclidean magnitude of the vector z, i.e.

$$|z| := \sqrt{(\operatorname{Re} z)^2 + (\operatorname{Im} z)^2}.$$

A complex number of modulus 1 is called *unimodular.* From basic trigonometry, any unimodular number takes the form

$$e^{i\theta} := \cos(\theta) + i\,\sin(\theta)$$

for some real number θ.

- **Polar form.** A nonzero complex number is in *polar* form if it is written as $z = r\,e^{i\theta}$, where $r > 0$ and θ are real. The number θ, which is unique only up to multiples of 2π, is called an *argument* of z, denoted $\arg z$; the unique value $-\pi < \theta \leq \pi$ is called the *principal argument*, denoted $\arg_p z$. Thus, every nonzero complex number can be written in polar form as

$$z = |z|\,e^{i\,\arg z}.$$

- **Conjugates.** The *conjugate* of a complex number z is its reflection in the real axis, i.e. the complex number

$$\overline{z} := (\operatorname{Re} z) - i(\operatorname{Im} z) = re^{-i\theta}.$$

Observe that

$$|z|^2 = z\,\overline{z}, \qquad \operatorname{Re} z = \frac{z + \overline{z}}{2}, \qquad \operatorname{Im} z = \frac{z - \overline{z}}{2i}, \qquad \overline{z + w} = \overline{z} + \overline{z}, \qquad \overline{z\,w} = \overline{z}\,\overline{w}.$$