Introduction to Geometric Langlands III (D. Nadler)

(Freiburg, 1 Aug 2012)

Reminders:

G/B - flag variety

Gr - LG/L^+G $\qquad \mathcal{F}\ell = LG/I$

Schubert stratifications:

$B \circlearrowright G/B \qquad L^+G \circlearrowright Gr \qquad I \circlearrowright \mathcal{F}\ell$

orbits: $\quad W \qquad \Lambda_G^{dom} \qquad W^{aff}$

microlocal stalks: $M_{x,\xi}\, F$

- M: stalk at x of vanishing cycles
- x: pt
- ξ: covector

$f : X \to \mathbb{C}$

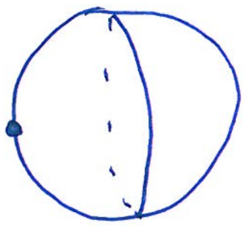
$f(x) = 0$

<u>example</u> $G = SL_2$

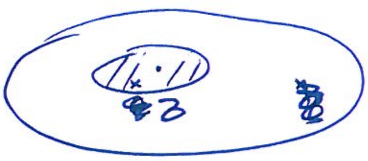
$P_S(\mathbb{P}^1)$

↑

Schubert stratification



First consider $P_S(\mathbb{A}^1)$



$$M_{0,z}(F) \underset{w}{\overset{\partial}{\rightleftarrows}} F|_z[-1]$$

$$M_{0,\varepsilon}(F) = \Gamma((D, \{\varepsilon\}), F)$$

$w \circ \partial$ = 1 - monodromy of F at z

$\partial \circ w$ = 1 - monodromy of vanishing cycles

Next $P_S(\mathbb{P}^1)$

$$P_S(\mathbb{P}^1) = \mathrm{Rep}\left(\begin{tikzcd} \bullet \arrow[r, bend left, "\partial"] & \bullet \arrow[l, bend left, "w"] \end{tikzcd} ;\ \begin{matrix} 1-\partial w,\ 1-w\partial \text{ invertible} \\ \cancel{\text{and}}\ w\cdot\partial = 0 \end{matrix} \right)$$

Indecomposables

$\bullet\ \mathbb{C}$

$\bullet\ \mathbb{C}$

$$\begin{tikzcd} \mathbb{C} \arrow[d, bend right] \\ \mathbb{C} \arrow[u, bend right] \end{tikzcd} \quad + \text{ dual}$$

$$\begin{tikzcd} \mathbb{C} \arrow[d, bend right] \\ \mathbb{C}\oplus\mathbb{C} \arrow[u, bend right] \end{tikzcd}$$

$\uparrow$ not B-equivariant

Def $\mathcal{H}_G = Sh(B\backslash G/B)$

$\mathcal{H}_{LG} = Sh(I\backslash LG/I)$

$\mathcal{H}^{sph}_{LG} = Sh(L^+G\backslash LG/L^+G)$

Convolution

$$\begin{tikzcd} X \arrow[d] \\ Y \end{tikzcd} \qquad \begin{tikzcd} & X\times_Y X \arrow[dl, "p_1"'] \arrow[dr, "p_2"] & \\ X & & X \end{tikzcd}$$