P. Fiebig Representations at critical level for affine Kac-Moody algebras

$\hat{\mathfrak{g}}$ - affine Kac-Moody algebra, $D = t\frac{d}{dt} \in \hat{\mathfrak{g}}$

$\hat{\mathcal{O}}$ - associated category $\mathcal{O}$

$\mathbb{C}K \hookrightarrow \hat{\mathfrak{g}}$ central line $\hat{\mathcal{O}} = \bigoplus_{k \in \mathbb{C}} \hat{\mathcal{O}}_k$

$K = k \cdot id$

$crt \in \mathbb{C}$ critical level

usual normalization, $crit = -h^\vee$, $L(-\rho) \in \hat{\mathcal{O}}_{crit}$

Z = Feigin - Frenkel center, $Z_{>0}$, $Z_{<0} \subseteq Z$

$\bar{\mathcal{O}}_{crit} = \hat{\mathcal{O}}_{crit}$ the restricted category $\mathcal{O}$

$\bar{\mathcal{O}}_{crit} = \{ M \in \hat{\mathcal{O}}_{crit} \mid Z_{\neq 0} M = 0 = Z_{<0} M \}$

Known : $\hat{\mathfrak{h}} \subseteq \hat{\mathfrak{g}}$ affine Cartan, $\hat{\mathfrak{h}}^*_{crit} = \{ \lambda \in \hat{\mathfrak{h}}^* \mid \lambda(K) = crit \}$

$\cdot\ \lambda \in \hat{\mathfrak{h}}^*_{crit} \Rightarrow L(\lambda) \in \bar{\mathcal{O}}_{crit}$

$\Rightarrow \bar{\Delta}(\lambda)$ restricted Verma

character formula known

$\hat{\mathcal{O}}_{crit} \circlearrowleft T = \otimes L(\delta)$ ('grading' functor)

$$ch\, \bar{\Delta}(\lambda) = e^{\lambda} \prod_{\substack{\alpha \text{ +ve} \\ \text{real} \\ \text{roots}}} (1 + e^{-\alpha} + e^{-2\alpha} + \cdots)$$

joint w/ Arakawa:

- $\overline{\mathcal{O}}_{crit}$ contains projective covers $\overline{P}(\lambda) \twoheadrightarrow L(\lambda)$
- $\overline{P}(\lambda)$ admits a restricted Verma flag.
- BGG-reciprocity: $[\overline{P}(\lambda) : \overline{\Delta}(\mu)] = [\overline{\Delta}(\mu) : L(\lambda)]$
- $\overline{\mathcal{O}}_{crit} = \bigoplus_{\Lambda} \overline{\mathcal{O}}_{\Lambda}$, $\Lambda = \{\lambda \in \hat{\mathfrak{h}}^*_{crit} \mid L(\lambda) \in \overline{\mathcal{O}}_{\Lambda}\}$

indecomposable block

is a $\hat{W}_{\Lambda}$-orbit

(linkage principle)

corresponding integral Weyl group

<u>Problem</u>: describe $\overline{\mathcal{O}}_{\Lambda}$, i.e., describe $P_{\Lambda} \subseteq \overline{\mathcal{O}}_{\Lambda}$

subcategory of projectives

<u>Aim</u>: Establish a functor

$$\left\{ \begin{matrix} \mathbb{C}\text{-parity sheaves} \\ \text{on affine Grassmanian} \end{matrix} \right\} \longrightarrow P_{\Lambda}$$

regular and integral

'parity' here = direct sums of IC-complexes on Schubert varieties.

<u>Lusztig</u> anticipated a similarity

critical level rep. theory

$\rightsquigarrow$ modular rep. theory

Feigin - Frenkel - Lusztig conjecture

$[\bar{P}(\lambda) : \bar{\Delta}(\mu)]$ = periodic polynomial evaluated at 1.

Similar to:

char $k > 0$

$\left\{\begin{array}{l}\text{k-parity sheaves}\\ \text{on affine Grassmannian}\end{array}\right\} \longrightarrow \left\{\begin{array}{l}\text{projective}\\ G_1T\text{-modules}\end{array}\right\}$

$\downarrow$ $\approx$ "local hypercohomology" $\to$ IW

$\left\{\begin{array}{l}\text{sheaves on}\\ \text{affine moment graph}\end{array}\right\}$ $\longrightarrow$ AJS - category $\longleftarrow$ ≀

IV (F, 2007) strange functor

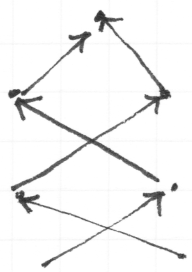
$\hat{\mathcal{O}}_{s,\Pi}$ verma

(deformed, positive level)

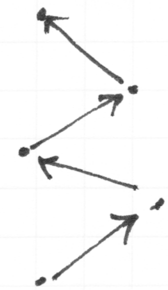
$\downarrow$ version of translation functor $\dfrac{\text{translation}}{(- \otimes L(\nu)^*)^{<\chi}}$

$\bar{\mathcal{O}}_{s,\Lambda}$

$\hat{\mathcal{O}}_{s,\pi}$ verma



$\vec{\mathcal{O}}_{s,\Lambda}$



sl_3 - picture identify affine weyl group with alcoves

antidominant weyl chamber

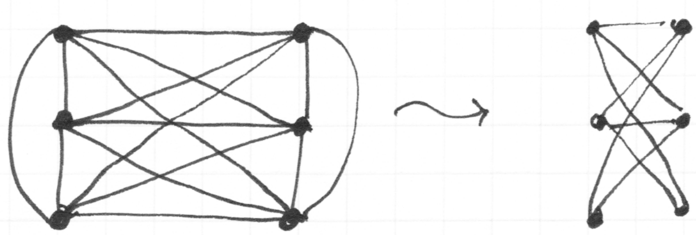
order matches up

<u>Lusztig</u> parabolic polynomials stabilize to generic polynomials

<u>Kato</u> Σ generic polynomials = (periodic polys).
(Kostant partition ~~from~~ function)

parabolic affine sl_2

(stable graph)



<u>Thm</u> (Lanini)

$$f^*(\mathcal{B}_{\mathbb{A}}^{stable}) \simeq \mathcal{B}_{\mathbb{A}}^{stable}$$

Braden-MacPherson sheaf

<u>conjecture</u> $\mathcal{B}_\Delta^{\text{stable}}$ has a canonical filtration w/ subquotients with periodic polynomials

$\mathcal{B}_\Delta^{\text{stab}}$ have verma flag

$\Rightarrow$ correspond to objects in $\hat{\mathcal{O}}_{s,\Gamma}$