Barr-Beck theorem

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Adjoint functors

f^* left adjoint to f_*

$\varepsilon : f^* f_* \longrightarrow id$ (counit) $\qquad \eta : id \longrightarrow f_* f^*$ (unit)

A monad on a category $\mathcal{C}$ is an endofunctor $T : \mathcal{C} \longrightarrow \mathcal{C}$ equipped w/ natural maps $\mu : T^2 \longrightarrow T$ and $\eta : id \longrightarrow T$ satisfying evident compatibilities.

Dual notion: comonad

An algebra over the monad T is an object $M \in \mathcal{C}$ equipped w/ evident maps satisfying evident compatibilities

The definition of a monad should be clear from:

example $\quad f^*$ left adjoint to f_*, now ~~to~~ take $T = f_* f^*$ and use unit/counit to define the maps.

Thm

Every monad arises from an adjunction.

There is also a suitable uniqueness statement here

consider $\quad \mathcal{C} \underset{f^*}{\overset{f_*}{\rightleftarrows}} \mathcal{D}$ $\quad$ want to describe $\mathcal{C}$ in terms of $\mathcal{D}$ + monad.

look up: ~~&~~ treatment of descent via this perspective. //