stable ∞-categories (D. Platt)

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Motivation

$\mathcal{A}$ - abelian category (too rigid)

$\downarrow$

$D(\mathcal{A})$ - derived category (triangulated)

Δ structure, not a property!

several problems w/ triangulated category form-alism

⇝ cones not ~~form~~ functorial

⇝ functor category not triangulated

stable ∞-categories

$\mathcal{C}$ - ∞ category.

$0 \in \mathcal{C}$ is a zero object if $\mathrm{Map}_{\mathcal{C}}(0, X)$ and $\mathrm{Map}_{\mathcal{C}}(X, 0)$ are contractible for all $X \in \mathcal{C}$.

If $\mathcal{C}$ has a zero object, we say $\mathcal{C}$ is <u>pointed</u>

A triangle in $\mathcal{C}$ is a map $\Delta^1 \times \Delta^1 \to \mathcal{C}$

$$\begin{array}{ccc} X & \to & Y \\ \downarrow & & \downarrow \\ 0 & \to & Z \end{array}$$

s.t. $f: X \to Y$ $g: Y \to Z$

$h: gf \sim 0$

⇝ more definitions

⇝ stable ∞-category