

Introduction to ∞ -categories

(P. 400)

13 Aug 2012

①

ordinary categories : $\text{Hom}(X, Y)$ is a set

∞ -categories : $\text{Hom}(X, Y)$ is a topological space

Simplicial sets

A simplicial set is a functor

$$\Delta^{\text{op}} \rightarrow \text{set}$$

\uparrow

simplex category : objects - $[n] = \{1, \dots, n\}$
morphisms - non-decreasing maps

from baby topology :

$$\text{Sing} : \text{Top} \rightarrow \text{simp. sets}$$

This has a left adjoint called geometric realization

(just follow your nose to construct it).

∞ -categories

example fix a topological space X .

Objects : points of X

1-mor : paths

2-mor : homotopies between paths

3-mor : " " homotopies

\vdots

\vdots

} note : nothing
is associative
on the nose