Completing the Square

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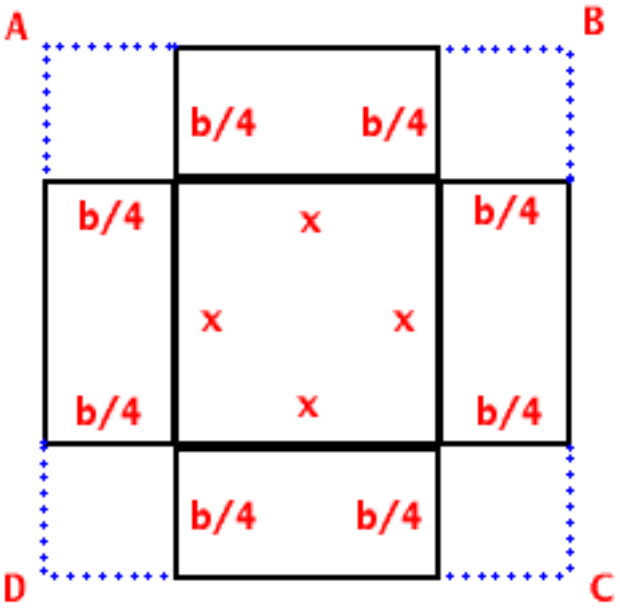
Consider the general quadratic equation $a_1x^2 + b_1x + c_1 = 0$, finding a solution to this equation is a equivalent to finding a solution to $x^2 + bx = c$ (in the original equation move the constant term to the other side and divide through by the coefficient of the x^2 term).

Consider a square of side x. Now on each of it sides consider a rectangle of width $\frac{b}{4}$ and length x.



Now consider the larger square $ABCD$ as indicated in the figure. This square has side of length $x + \frac{b}{2}$ and hence and area of $(x + \frac{b}{2})^2$, but if we add up the areas of all the squares and rectangles that make up $ABCD$ we also have that its area is $x^2 + 4\frac{b}{4}x + 4\left(\frac{b}{4}\right)^2$. This along with the fact that $x^2 + bx = c$ gives us

$$\left(x + \frac{b}{2}\right)^2 = c + \frac{b^2}{4}$$

solving for x we have

$$x = \sqrt{c + \frac{b^2}{4}} - \frac{b}{2}$$