

The passage from IF to C via CBB

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(Freiburg, Jul 2012)

①

$f: X \rightarrow Y$  proper algebraic map of varieties /  $\mathbb{C}$

$\mathcal{F}$  - simple perverse sheaf on  $X$  of geometric origin  
 $\Rightarrow Rf_* \mathcal{F}$  perverse semi-simple complex on  $Y$

w/ coefficients in

or just assume we are in this situation a field of char 0.

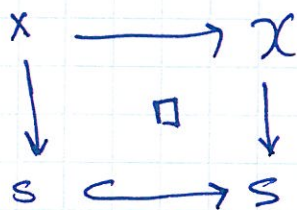
1) without loss of generality we can assume  $X, Y, \mathcal{F}$  is defined over  $A \subset \mathbb{C}$

$\uparrow$  finite type /  $\mathbb{Z}$

$X/A$ , maybe  $X = \mathbb{P}^n(\mathbb{C})$ ,  $\sum a_\alpha x^\alpha = 0$   $a_\alpha \in \mathbb{C}$

$\uparrow$  defined over

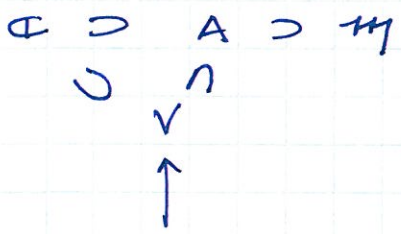
$$S = \text{spec}(\mathbb{Z}[a_\alpha])$$



$\exists$  stratification of base, s.t. topological type of fibres is constant on strata.

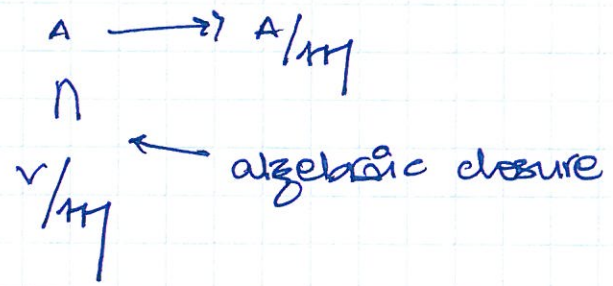
$X/A$  - fix stratification  $\overline{T}$ , fix local systems on strata (finitely many)  $\mathcal{L}$ 's.

(2)

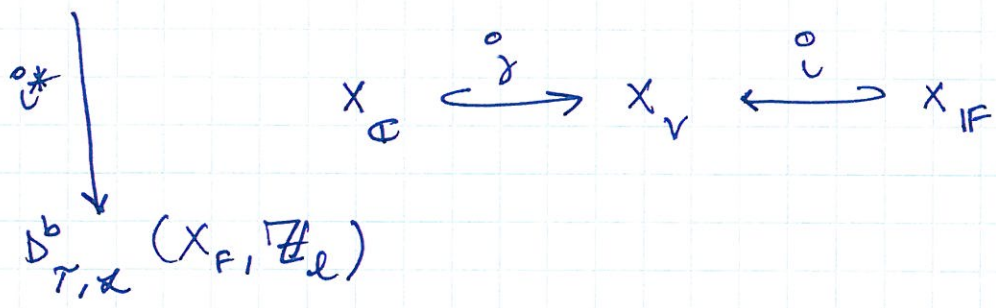


DVR, strictly henselian

means to be contractible in the étale topology



$$D_{T, \mathbb{Z}}^b(X_V, \mathbb{Z}_\ell) \xrightarrow{\circ j^*} D_{T, \mathbb{Z}}^b(X, \mathbb{Z}_\ell)$$



$\pi: Y \rightarrow X$  semi-small proper

$\uparrow$   $\text{codim} \{x \in X \mid \dim_{\mathbb{C}} \pi^{-1}(x) > n\} \geq 2n$

assume smooth

$\Rightarrow R\pi_* \underline{Y}[d_Y]$  is perverse

$\leftarrow$  as the stalks of are cohomology of fibres

now use the semi-small assumption.

$$\rightsquigarrow R\pi_* \underline{Y}[d_Y] = \bigoplus IC_S$$

$\uparrow$

the skyscraper pieces will be ones where fibres are of max dim.