Usual conventions: all functors are derived; sheaves have coeff-icients in a reasonable ring, etc.

$\underline{X}$ - constant sheaf on X

$v: \mathbb{R}_{\geq 0} \hookrightarrow \mathbb{R}$

$$\{0\} \xhookrightarrow{\mathring{i}} \mathbb{R}_{\geq 0} \xhookleftarrow{\mathring{j}} \mathbb{R}_{>0}$$

<u>Prop. 1</u> $\quad v^! \underline{\mathbb{R}} \simeq \mathring{j}_! \underline{\mathbb{R}_{>0}}$

<u>proof</u> Apply the localization triangle $\mathring{j}_!\mathring{j}^* \to \mathrm{id} \to \mathring{i}_*\mathring{i}^* \xrightarrow{+1}$ to $v^!\underline{\mathbb{R}}$ to see that it suffices to show $\mathring{i}^* v^! \underline{\mathbb{R}} = 0$.
Let $\pi: \mathbb{R} \to \mathbb{R}_{\geq 0}$ be the evident contraction. By the usual homotopy argument $v^!\underline{\mathbb{R}} \simeq \pi_! \underline{\mathbb{R}}$. Now $\mathring{i}^*\pi_!\underline{\mathbb{R}} \simeq \tilde{\pi}_! \underline{\mathbb{R}_{\leq 0}}$, where $\tilde{\pi}: \mathbb{R}_{\leq 0} \to \{0\}$. But $\tilde{\pi}_! \underline{\mathbb{R}_{\leq 0}} = H^*_c(\mathbb{R}_{\leq 0}) = 0$.
Here the last vanishing follows by devissage $\{0\} \hookrightarrow \mathbb{R}_{\leq 0} \hookleftarrow \mathbb{R}_{<0}$.

QED

Write ω_X for the dualizing complex on X.
Let $\bar{\Omega}$ be a manifold w/ boundary, and let $\Omega \xhookrightarrow{\mathring{j}} \bar{\Omega}$ be the interior.

<u>Prop 2</u> $\quad \omega_{\bar{\Omega}} \simeq \mathring{j}_! \omega_{\Omega}$

<u>proof</u> Apply Prop 1.

QED

<u>Remark</u> Prop 2 might be inaccurate as stated (orientations? local nature of ω_X?). However, in practise (?) we are only using it inside $\mathbb{R}^n$ w/ a half space. So bit of a thankless task to verify all the details.